Spectral Analysis and Filtering of Deformation Monitoring Data

presented at

Surveying and Geomatics Educators Society (SaGES)
2025 Biennial Conference
June 15-19, 2025
Vancouver, Canada

by

John Ogundare, PhD., P.Eng.
Instructor, British Columbia Institute of Technology (BCIT), Burnaby

Objectives

For students to:

1. Gain experience in deformation data analysis:
   - Determining true deformation trend (syllabus topics: curve fitting and modeling)
   - Determining possible periodic components of deformation data (syllabus topic: frequency domain analysis)
   - Correlate possible periodic components with seasonal precipitation and temperature variations of the monitoring regions.
2. Understand the properties of time-domain (Moving Average) filtering of monitoring data – compared with Fourier filtering

Introduction and Problem Definition

- Deformation data (**DD**) collected over a period at a point can be represented by

$$\mathbf{DD} = \mathbf{DT} + \mathbf{PC} + \mathbf{GWN} \qquad (1)$$

where **DT** = deterministic trend, **PC** = periodic component, **GWN** = Gaussian white noise

$$\mathbf{DT} = a \times t + b \qquad (2)$$

where a = deformation rate (of interest), t = time, b = intercept

- Let Trend residuals (**TR**) be given as **TR = DD – DT** or:

$$\mathbf{TR} = \mathbf{PC}(\text{dependent noise}) + \mathbf{GWN}(\text{independent noise}) \qquad (3)$$

- **Problem:** Are the determined **DT** and **PC** reasonable and complete, representing the actual phenomenon being studied (with the assumption of **GWN**)?

Test Monitoring Data Sets

- Our test monitoring data sets are from two sites:

Metro-V (16 monthly data): The subsidence monitoring data relating to a **bridge in Metro Vancouver**, BC, from February 2022 and June 2023 (done in 16 monthly epochs). Four vertical monitoring object points **Pt. 500** (in the northern section, closer to construction site), **Pt. 503** (southern section), **Pt. 516** and **Pt. 514** (in the middle section) of the bridge are considered in this study. These points are built-in bolts set into the concrete piers (higher in the south) above the ground level.

Van-I (19 monthly data): Subsidence monitoring of ground points in **southern Vancouver Island**, BC. The points consist of steel pipes (1.5” in diameter) installed into the earth, protruding from between 1- 2 m. A clamp was fixed to the pipe roughly 30-50 cm above the ground upon which the leveling rod were placed. Monitoring occurred weekly (same day each week), but interpolated monthly values were considered for our analysis. Three points considered: **BN**, **BL** and **GR**.

Approach: Iterative Trend Modeling with Spectral Analysis (1/3)

- The iterations (by trial and error) are done to ensure that periodic components are correctly identified from the subsidence data - when all points fall within the constructed confidence bounds

1. Perform deterministic trend modelling (using polynomial fit) of deformation data (**DD**) with missing data interpolated using quadratic spline functions.
    - <u>Significance of coefficients</u>: Check if fitted coefficients fall within 95% confidence bounds, indicating that there is 95% chance that new observations are contained within the lower and upper prediction bounds
    - Check goodness of fit of trend models – **RMSE must improve in trend fitting from one iteration to another**

Approach: Iterative Trend Modeling with Spectral Analysis (2/3)

2. Remove identified deterministic trend from the deformation data (**DD**) to obtain Trend Residual (TR0):
   - Statistically analyse TR0 to determine if they are Gaussian white noise (distributed as Gaussian with mean of zero, constant variance, and uncorrelated in time)-
     - Zero mean – achieved in scatter plots
     - Constant variance at all points in time – not achieved in scatter plot (difficult to achieve because of possible false fluctuations)
     - Bell shape patterns –Chi-square goodness of fit (useful in localizing significant periodic components); others (histograms, normal probability plots) are difficult to interpret
   - Perform autocorrelation function (acf) analysis - useful in identifying non-random patterns among residuals. TR0 are insignificant if all the acf values fall within the confidence intervals CI(τ) at (1-α)100% level (with α as significance level), given as:

$$CI(\tau) = \pm z_{1-\alpha/2} \times se(\tau) \quad (4)$$

where $z_{1-\alpha/2}$ is the z-score and the standard error for n samples at lag τ is

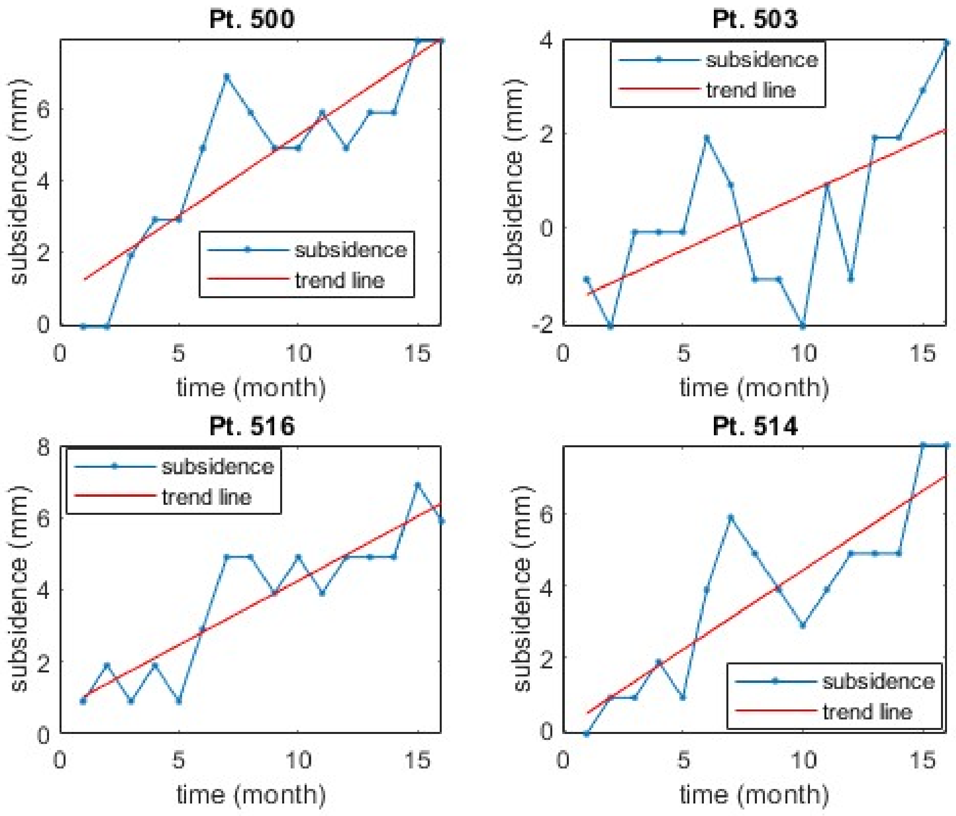
$$se(\tau) = \frac{1}{\sqrt{n-1}} \quad (5)$$

Approach: Iterative Trend Modeling with Spectral Analysis (3/3)

3. **Perform Iteration**: Fourier analysis and iterative update of **DD** & trend models
   a. Perform Fourier decomposition of TR0 to identify significant frequency components.
   b. Remove the significant frequency components from the **DD** to obtain **filtered deformation data** (**FDD1**).
   c. Perform deterministic trend modelling of **FDD1** to obtain **Trend1 + statistics**;
   d. Remove **Trend1** from **FDD1** to obtain Trend Residual Iteration 1 (**TR1**) + **statistics**
   e. Repeat steps 2 and 3 for another iteration, if necessary, until statistical analysis indicates that the trend residuals at that iteration are Gaussian white noise (mainly based on the autocorrelation function (acf) plots analysis).
4. Summarize the final trend and the significant periodic components identified from **DD**.

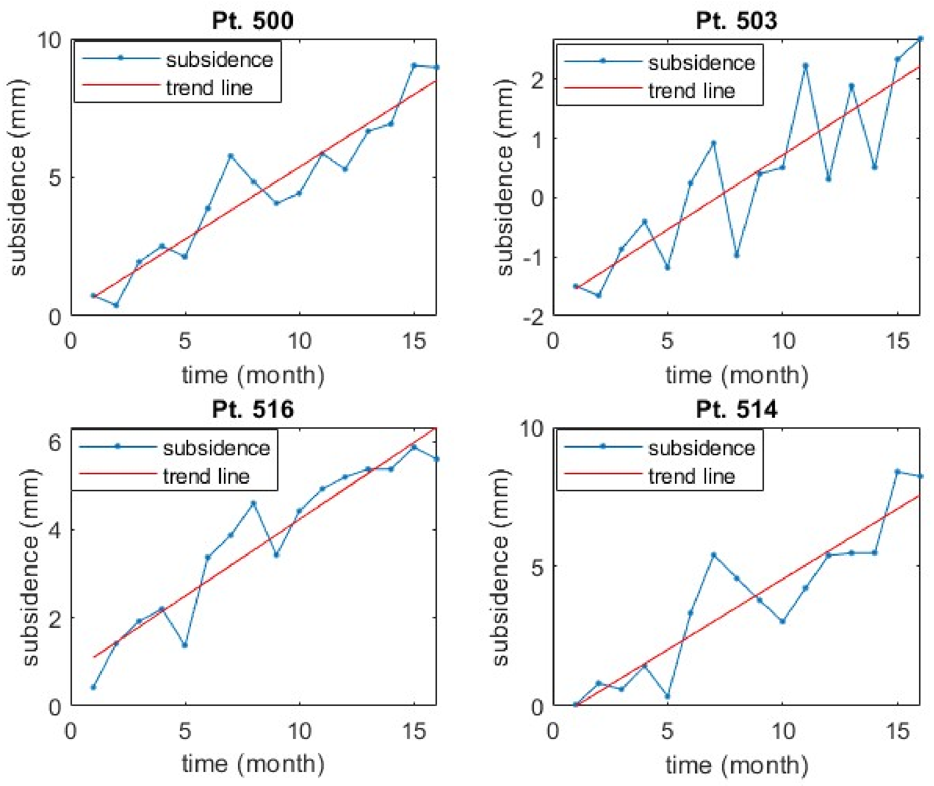
(DD = Deformation Data)

Deterministic Trend Analysis: Metro-V



Original trend lines (based on original DD).

DD = Deformation Data.



Final iteration (based on updated DD)

[Residuals are more symmetric about zero; but variance is not constant (possible other components)].

Computed Trend values (Uplift per month) with 95% Confidence Bounds: Metro-V

Station	Original Trend : a (per month)	Final Trend: a (per month)
500	2.1 mm ± 0.7 mm	2.5 mm ± 0.5 mm
503	1.1 mm ± 0.8 mm	1.2 mm ± 0.4 mm
516	1.7 mm ± 0.5 mm	1.7 mm ± 0.3 mm
514	2.1 mm ± 0.7 mm	2.4 mm ± 0.6 mm

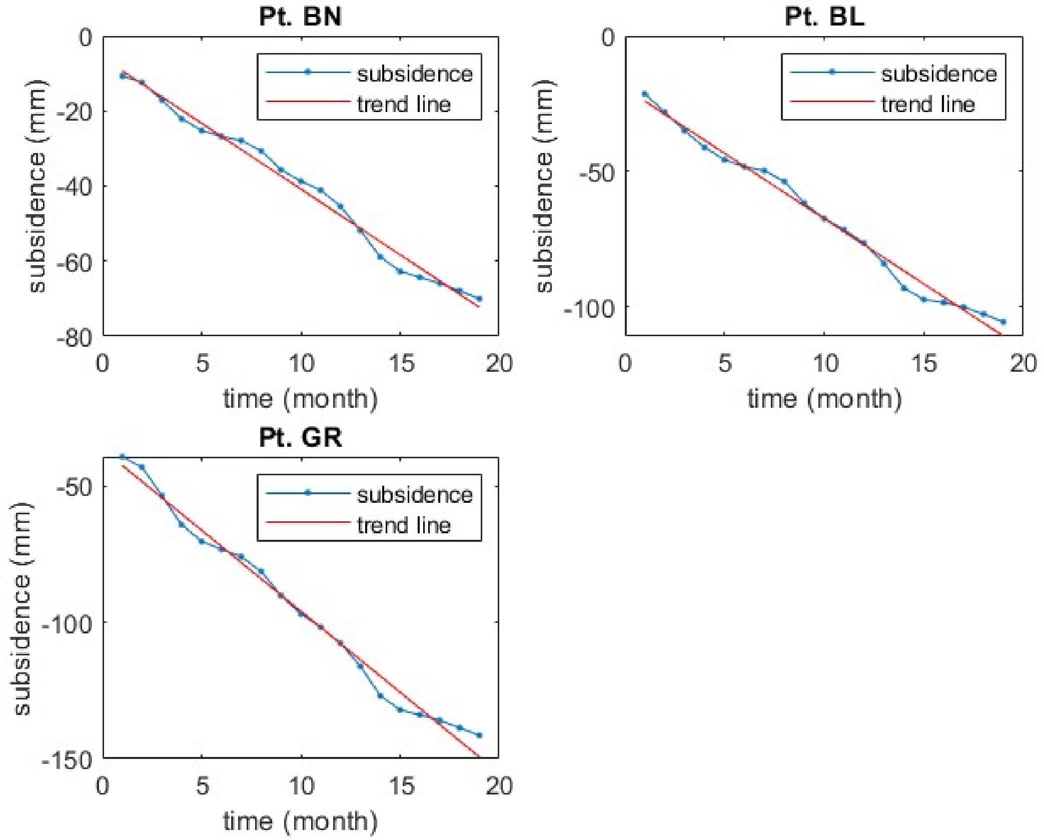
Note: Significant periodic components were first removed from the **DD** before the Final Trend values were determined.

The confidence bounds for trend rate (a):

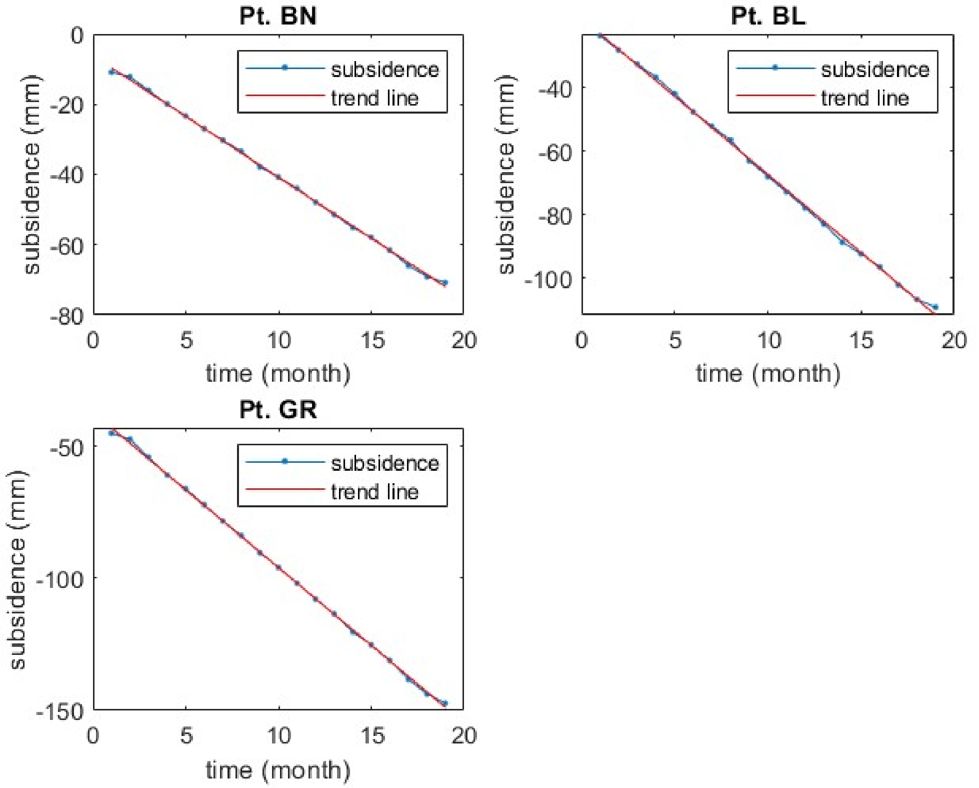
$$C = \pm t_{1-\frac{\alpha}{2}} S \tag{6}$$

where $t_{1-\frac{\alpha}{2}}$ is the Student's t distribution value with a significance level α, and S is the standard deviation of coefficient b determined in the least squares estimation.

Deterministic Trend Analysis: Van-I



Original trend lines (based on original DD).



Final iteration (based on updated DD)

Residuals are more symmetric about zero lines; but variance is not constant (possible other components).

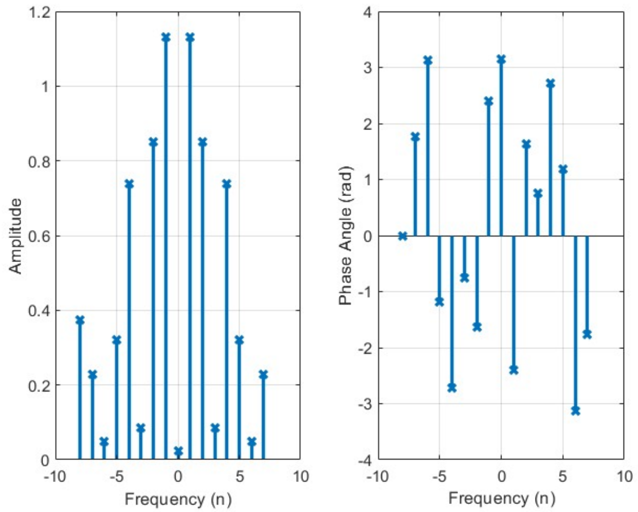
DD = Deformation Data.

Computed Trend values (Subsidence per month) with 95% Confidence Bounds: Van-I

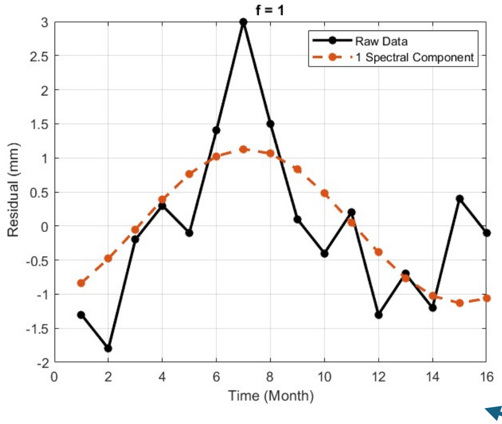
Station	Original Trend : a (per month)	Final Trend: a (per month)
BN	-19.7 mm ± 1.2 mm	-19.5 mm ± 0.3 mm
BL	-27.1 mm ± 1.6 mm	-27.7 mm ± 0.5 mm
GR	-33.4 mm ± 2.0 mm	-33.2 mm ± 0.4 mm

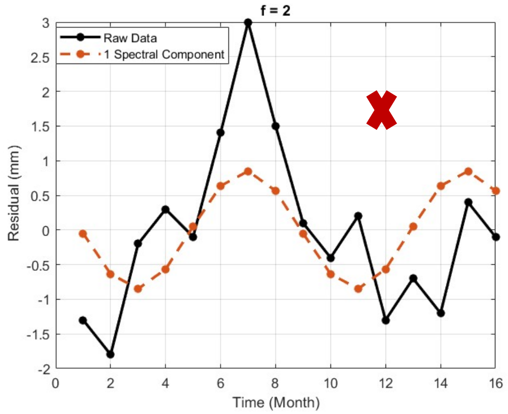
Note: Significant periodic components were first removed from the **DD** before the Final Trend values were determined.

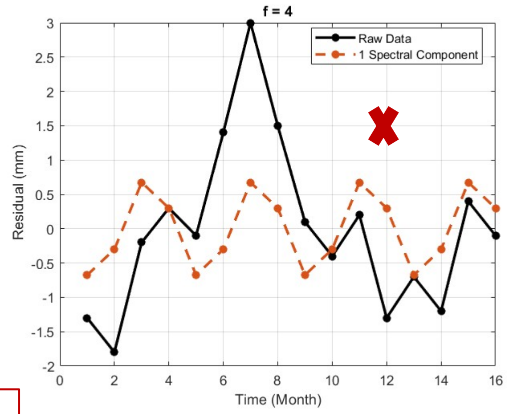
Example of Spectral Analysis (Metro-V): Pt. 500

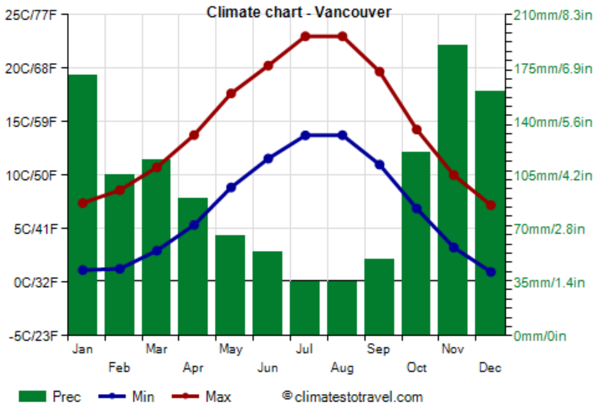


(Month 1 = March)









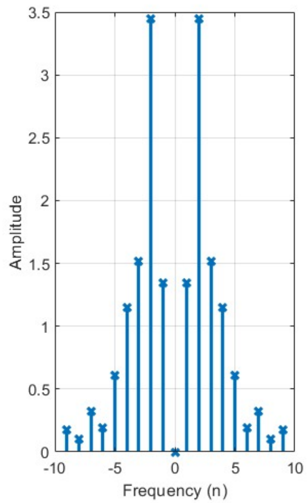
Annual average temperatures & precipitation in Vancouver

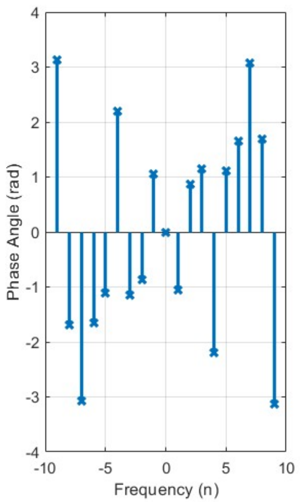
Note: Seasonal effects of only f = 1 (with highest spectral power) are significant according to ACF Analysis (with possible phase shift)

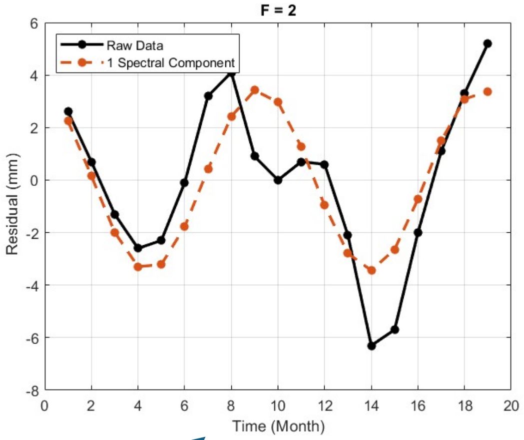
Pt.500 is on a low-lying pier close to construction site

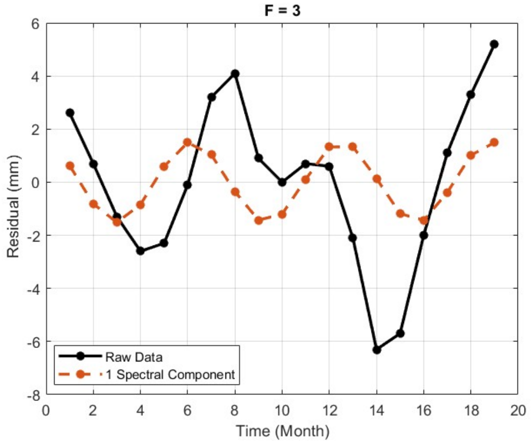
✘ False fluctuations eliminated.

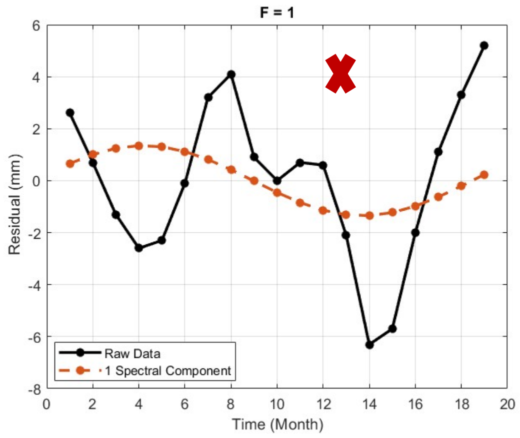
Example of Spectral Analysis (Van-I): Pt. BL



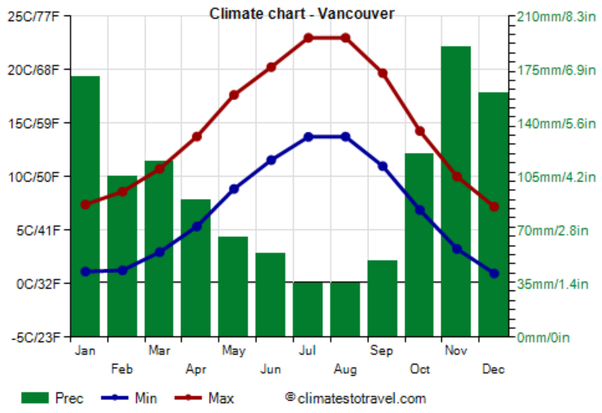




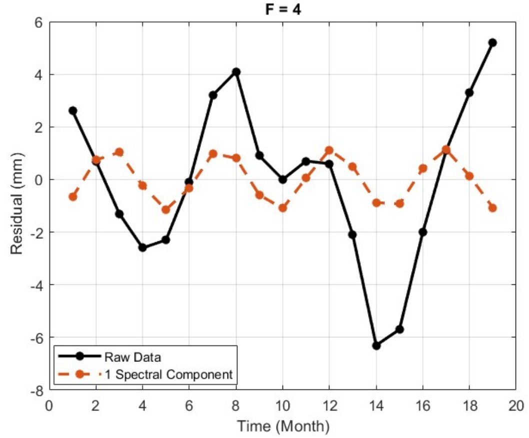


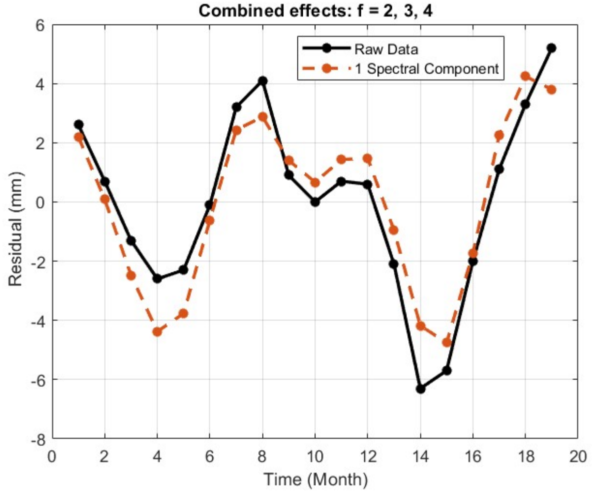


Note: Seasonal effects with f = 2 (highest spectral power), f = 3 (second), f = 4 (fourth) are significant according to ACF analysis



Annual average temperatures & precipitation in Vancouver





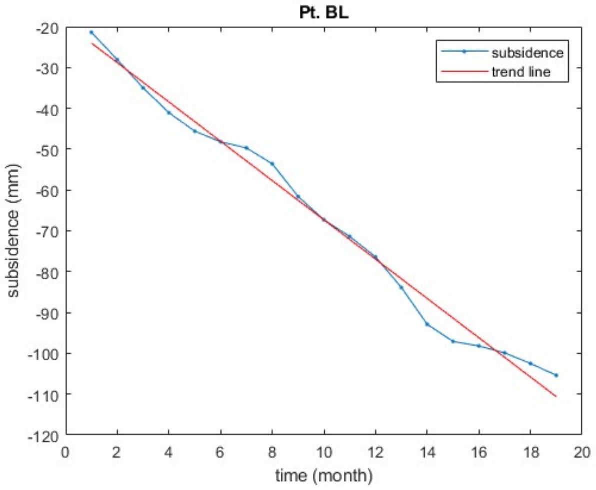
(Month 4 = January)

Time Domain Moving Average (MA) FIR Filtering

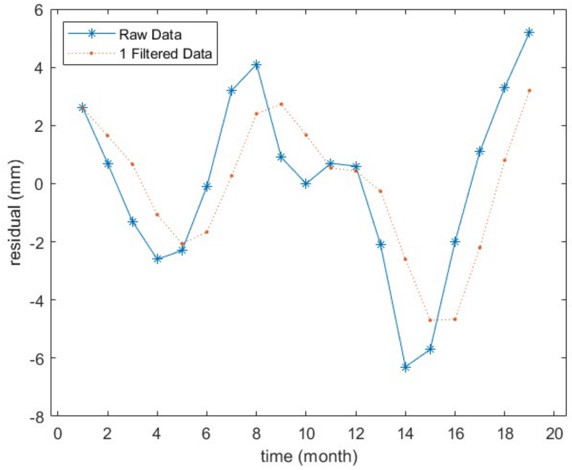
- Moving Average (MA), a type of Finite Impulse Response (FIR) filter is a low-pass filter – retains low frequency components of a signal while attenuating the intensity of the high frequency components (such as noise).
  - How it retains low frequency components when many components are involved
  - How it attenuates the intensity of high frequency components
- **Goal:** Understanding time domain filtering procedure of MA FIR filtering at Pt. BL:
  1. Perform MA filtering on TR0 based on the frequency values identified through Fourier analysis, e.g. f = 2 Hz (6-month averages); f = 3 Hz (4-month averages), f = 4 Hz (3-month averages) to obtain MA_FTR
  2. MA_FTR is the simple MA filtered data of TR0
  3. Perform Fourier decomposition of the MA_FTR to determine the significant frequency components (SFCs) retained by the MA filter.
  4. Summarize the SFCs retained by the MA filter and discuss the acf of MA_FTR minus SFC.

3-Month MA Filtering & Spectral Analysis of Residuals of Pt. BL

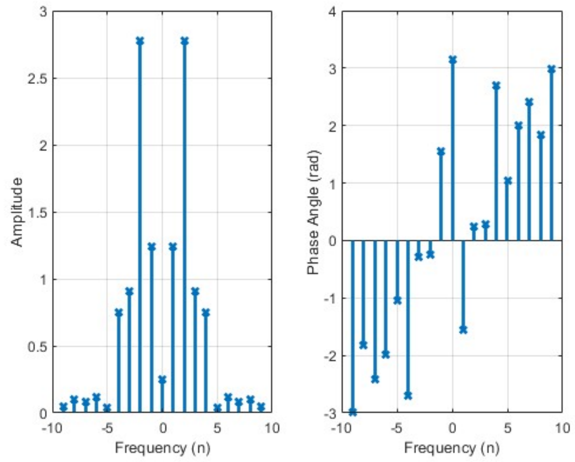
Trend fitting to original subsidence data.

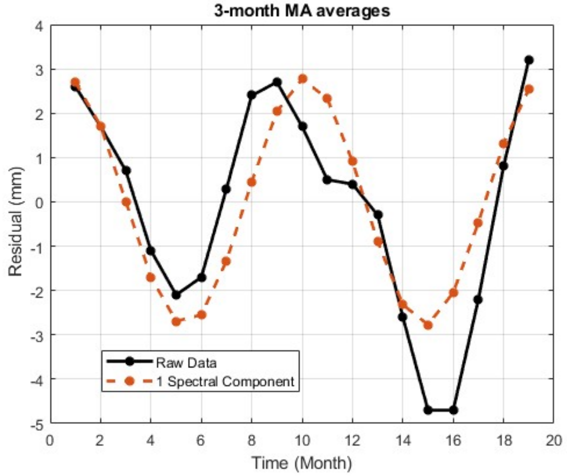


3-Month MA filtering of residuals.

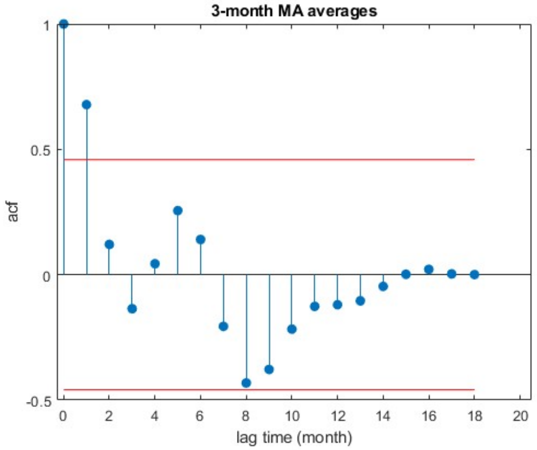


Spectral plot of 3-Month MA filtered data.





Significant frequency component (SFC) for 3-month MA filtered data (f = 2).

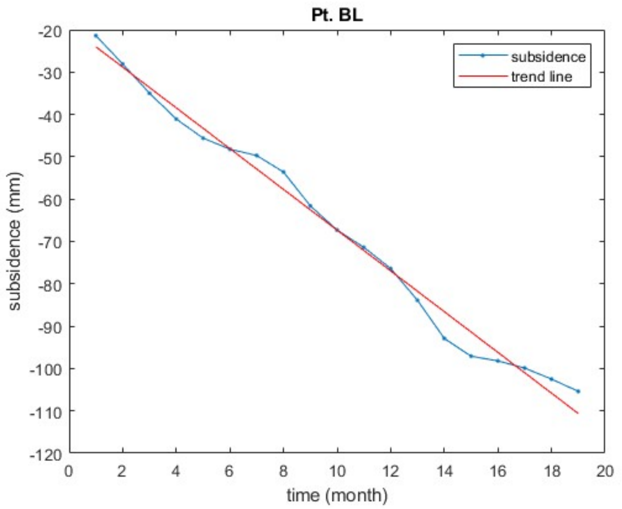


Acf plot of raw data minus SFC.

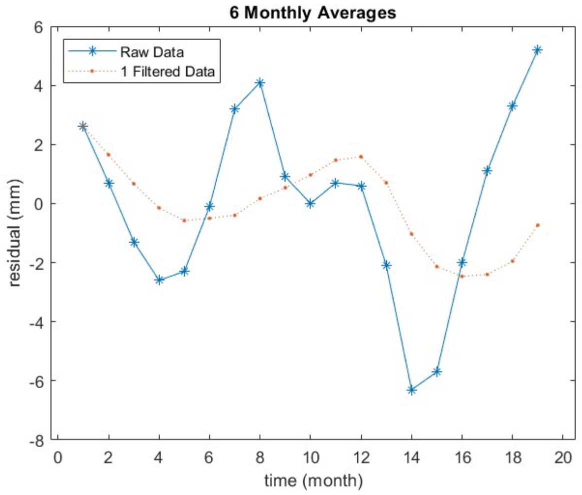
- Only **f = 2** (strong; 20% weakened);
- Others are weakened **f = 3** with 20% weakened & **f = 4** with 25% weakened
- MA_FTR are correlated according to acf

6-Month MA Filtered Data & Spectral Analysis of Pt. BL

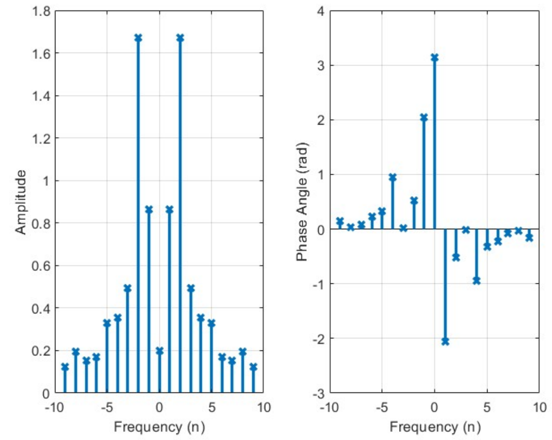
Trend fitting to original subsidence data.

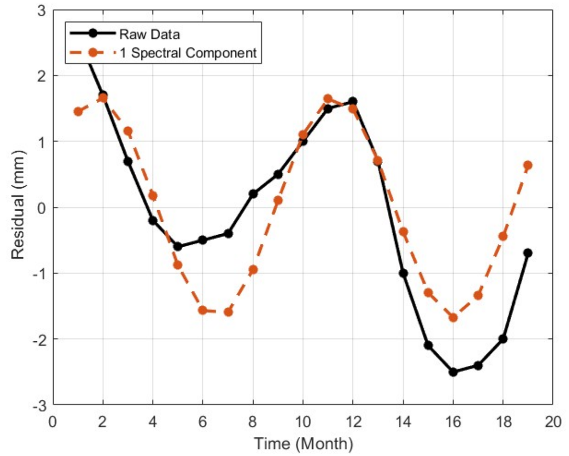


6-Month MA filtering of residuals.

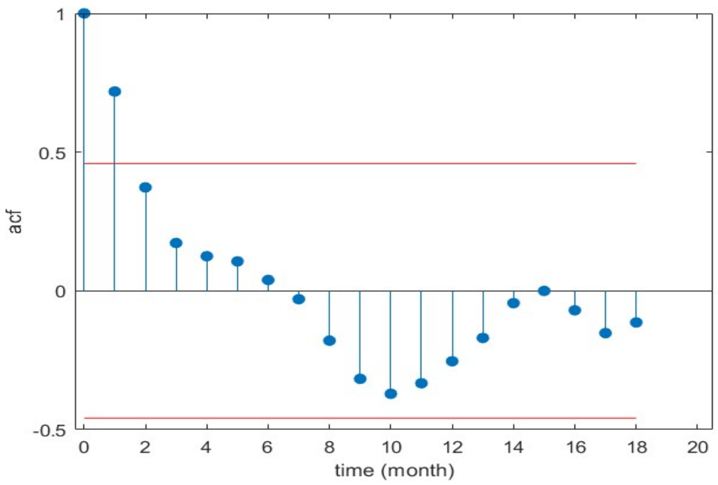


Spectral plot of 6-Month MA filtered data (f = 2).





Significant frequency component (SFC) for Filtered data (f = 2).



ACF plot of MA filtered data minus SFC.

- Only **f = 2** (strong; 50 % weakened);
- Others are weakened **f = 3** with 70% weakened & **f = 4** with 70% weakened
- MA_FTR are correlated according to acf

Conclusions

- Iterative trend modeling with spectral analysis provided:
  - Improved trend models which are more certain (smaller confidence bounds at 95% level)
  - Significant periodic components with the help of ACF
- Some of the periodic components match the annual temperature variations in Vancouver
- MA FIR filtering seems to have the following properties:
  - Retains weakened low frequency component as expected (with lower % of attenuation)
  - But also retains other higher frequency components which are weakened at higher % – confirmed to be present in the filtered data by acf analysis
  - Attenuation increases with increasing sizes of the filter windows, e.g., 6 months MA filtering attenuates by a factor two compared with 3 months MA filtering

END